DOCUMENT RESUME

ED 263 134 TM 850 604

AUTHOR Greeno, James G.; Johnson, Walter
TITLE Competence for Solving and Understanding Problems. 1985/17.
INSTITUTION Pittsburgh Univ., Pa. Learning Research and Development Center.
SPONS AGENCY National Inst. of Education (ED), Washington, DC.
PUB DATE 85
NOTE 17p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Arithmetic; Cognitive Processes; *Cognitive Structures; Elementary Secondary Education; Instruction; *Learning Strategies; *Models; *Problem Solving; *Process Education
IDENTIFIERS Conceptual Analysis; *Conceptual Models; *Process Analysis; Word Problems (Mathematics)

ABSTRACT

The relation of knowledge for solving problems to understanding of general principles in a subject-matter domain is discussed. Theoretical representations, called "conceptual competence," are presented to represent principles that are believed to be understood implicitly by individuals. Implications for problem-solving procedures can be derived from these principles. In a task for which process models have been developed, the process models are related to the formulation of general principles that are reflected in the process. (Author/PN)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

COMPETENCE FOR SOLVING AND UNDERSTANDING PROBLEMS

James G. Greeno and Walter Johnson
University of Pittsburgh

Learning and Research Development Center
University of Pittsburgh

1985

The material reported herein was supported by the Learning Research and Development Center, funded in part by the National Institute of Education, U.S. Department of Education. The opinions expressed do not necessarily reflect the position or policy of NIE, and no official endorsement should be inferred.

COMPETENCE FOR SOLVING AND UNDERSTANDING PROBLEMS

James G. Greeno and Walter Johnson

University of Pittsburgh

Pittsburgh, Pennsylvania

U.S.A.

The authors discuss the relation of knowledge for solving problems to understanding of general principles in a subject-matter domain. Theoretical representations, called "conceptual competence," are presented to represent principles that we believe are understood implicitly by individuals. Implications for problem-solving procedures can be derived from these principles. In a task for which process models have been developed, we relate the process models to the formulations of general principles that are reflected in the process.

An important development in the study of problem solving has been the analysis of processes and knowledge that are specific to a subject-matter domain. Detailed models, with supporting data, have been developed for solving problems in basic physics (Larkin, McDermott, Simon, & Simon, 1980), high-school geometry (Anderson, 1983; Greeno, 1978), and elementary arithmetic word problems (Briars & Larkin, 1984; Kintsch & Greeno, 1985; Riley, Greeno, & Heller, 1983) as well as procedures for calculation (Brown & Burton, 1978; Resnick, 1983). These models contrast with earlier analyses of problem solving, especially by Newell and Simon, which characterized general problem-solving methods, applied in situations where the problem solver has a minimum of experience or training.

The more recent studies of domain-specific problem solving have direct relevance to the psychology of education. A student's learning is often tested by presenting problems to the student. Models of the knowledge required for solving the problems, then, provide hypotheses about the knowledge that students acquire in order to succeed in the instruction they are given in courses. The processes and knowledge structures included in models of successful problem solving can be considered as objectives for instruction, in that they show a set of specific cognitive structures and procedures that can be the targets of instruction.

Indeed, models of problem solving in school subjects already are providing a basis for new instructional methods and materials. The development of a process model--especially if it is in the form of a computer program--often requires forming explicit hypotheses about some aspects of knowledge that usually remain tacit. Then, with those more specific ideas about knowledge that students should acquire, new instruction can be designed that focuses on the specific processes and structures that have been identified in the model.

These theoretical developments have provided clearer ideas about strategic knowledge, problem representation, and structural understanding, and they are significant advances in the psychology of problem solving and instruction. However, an important problem is not reached by these advances.

The problem is the relation of knowledge for solving problems to understanding of general principles in the subject-matter domain. The ability to solve problems or answer questions is fine, as far as it goes, but in almost all cases, we care more about understanding at a level that is conceptually deeper than the specific problems and questions that we use in learning exercises and texts. We are primarily concerned with whether students come to understand general concepts and principles.

The models of problem solving that have been constructed do not include representation of general principles of the domain. In the remainder of this paper, we describe a theoretical project in which we are exploring a way of characterizing understanding of general principles and its relation to knowledge used in solving problems.

We are working on theoretical representations that are called conceptual competence. We want to represent principles that we believe are understood implicitly by individuals. By implicit understanding we mean that the principles play a significant functional role in individuals' knowledge for problem solving, although individuals do not necessarily know how to state the principles or explain their significance. The principles are the basis for judging that procedures are correct. Principles include constraints on correct procedures as well as justifications for procedural components. Individuals can learn to perform correctly without understanding the underlying principles of correctness, but there are many cases in which there is evidence that individuals also have understanding. For example, in the domain of counting, Gelman and Meck (1983) have found that preschool children can distinguish between correct and incorrect counting done by a puppet, even when the puppet uses nonstandard correct procedures; and Gelman and Gallistel (1978) found that preschool children can generate modified counting procedures in response to special constraints. Such performance provides strong

evidence that children's knowledge is not restricted to "mechanical" cognitive procedures, but that they also know what makes the procedures correct--although this understanding is implicit.

In our analyses, we formulate the general principles of a domain in a way that allows us to derive their implications for procedures.

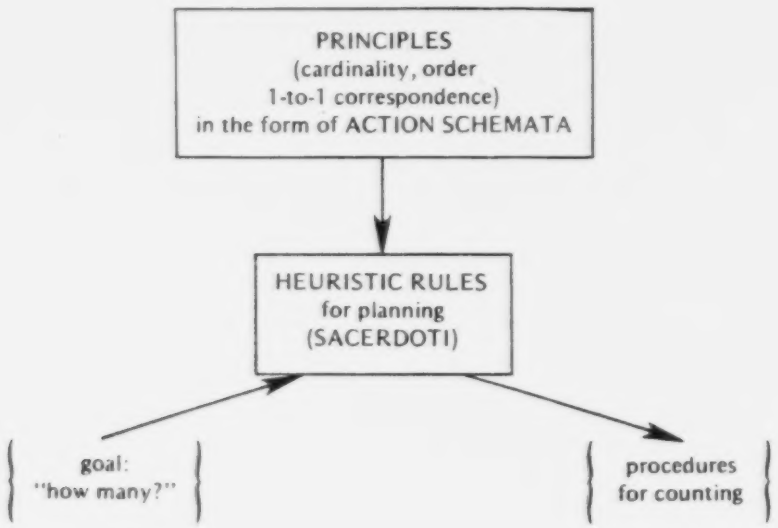

Figure 1: Components of a Competence Analysis

For example, in an analysis of counting, principles of cardinality, order, and one-to-one correspondence were formulated as schemata for actions, and a logic of planning was used to derive procedures of counting. The analysis shows how the procedures are constrained and justified by the principles.

The kind of theory used in these analyses is like the theory that Chomsky (1965) developed to represent implicit understanding of syntactic rules. Following Chomsky, we refer to our hypothesis as a form of competence. There are some important distinctions between our analysis and Chomsky's, which are discussed after presenting some of our results.

In the remainder of this paper, we consider a specific task, solution of word problems in elementary arithmetic. Some examples of the task are:

(1) Jay had eight books; then he lost three of them; now how many of them does he have?

(2) Four boys and nine girls are on the playground; how many boys and girls are there altogether?

(3) Kay has six apples and she has four oranges; how many more apples than oranges does she have?

Solution of problems like these includes understanding the text and performing a calculation. Models of these processes have been developed by several investigators, and the models differ in a theoretically interesting way.

In one class of models (Kintsch & Greeno, 1985; Riley, Greeno, & Heller, 1983), processes of language understanding construct quite complete representations, and the problem solutions are obtained with very simple knowledge of operators. In another class of models (Briars & Larkin, 1984), the representations formed from understanding the language are much less detailed, and the problem-solving operators include more sophisticated knowledge to distinguish the different problem situations that occur. We refer to the first class of models as being schema based, and the second class as being propositional, because the representations formed in the second class consist of quite simple propositions.

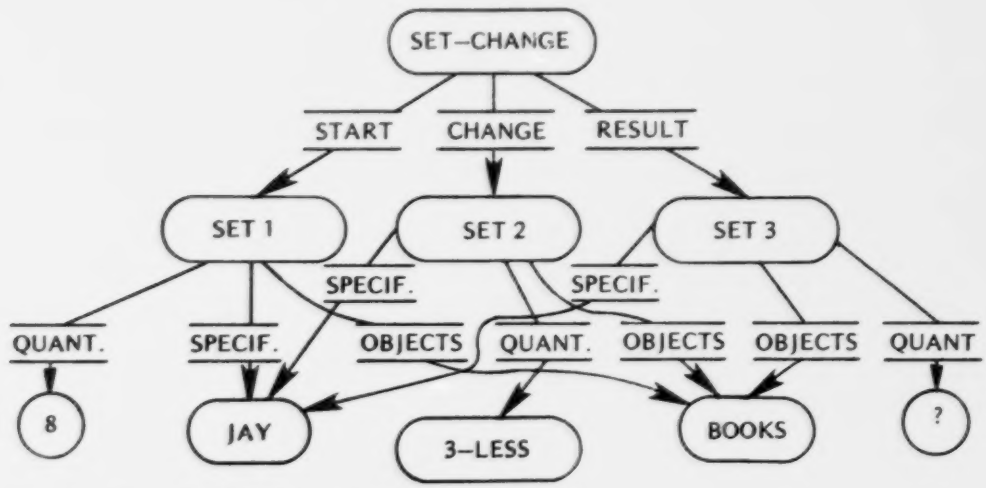

Figure 2: Schema-Based Representation of Problem (1)

For example, to solve problem (1) the schema-based model constructs a representation that is shown in Figure 2, as a semantic network. This is constructed using a schema in the model that represents changes in quantities--events that either increase or decrease the amount of some single quantity, such as Jay's books. The model uses a subschema that represents sets in a standard way, with the kind of objects in the set, a specification, and the quantity of the set. It also uses standard structures to represent relations among sets, in this case, one set is the start set, one is the change, and one is the result. The schema-based model uses different schemata for representing different kinds of relational structures. It has a schema of subset-set relations for representing combinations such as problem (2), and a schema of set differences for representing comparisons such as problem (3).

With a schematized representation such as Figure 2 shows, the solution of the problem can be obtained either with knowledge of arithmetic or by counting operations. The models that have been implemented simulate solutions by counting. For problem (1), the model counts a set of eight blocks, then takes three away from the set, and finds the answer by counting the blocks that remain.

PROPOSITIONAL MODEL:

"Had eight books"	Count and place eight blocks
"Lost three of them"	Count and take away three blocks
"Now how many"	Count blocks that remain

Figure 3: Solution of the Propositional Model to Problem (1)

The propositional model does not form a schematized representation of the problem, but only forms propositions corresponding to the text. These do not include symbols that refer to the sets of books, or to any relations among the sets. The propositions include cues that cause the model to perform actions. For example, "had eight books" triggers a procedure that counts eight blocks and puts them in a special place. "Lost three of them" causes the model to count three of the blocks in the special place and take them away. "Now how many" causes the model to count the blocks that are in the special place.

The difference between the models is not very significant on simple problems like problem (1). The difference is more interesting for more difficult problems.

(4) Jay had some books; then he lost three of them; now he has five of them left; how many books did Jay have in the beginning?

The schema-based model constructs a representation with three sets, like the one shown before, except that the quantity of the start set is unknown, and the change and result sets have known quantities. When the starting quantity is unknown, the schema-based model constructs a new set of relations involving sets and subsets.

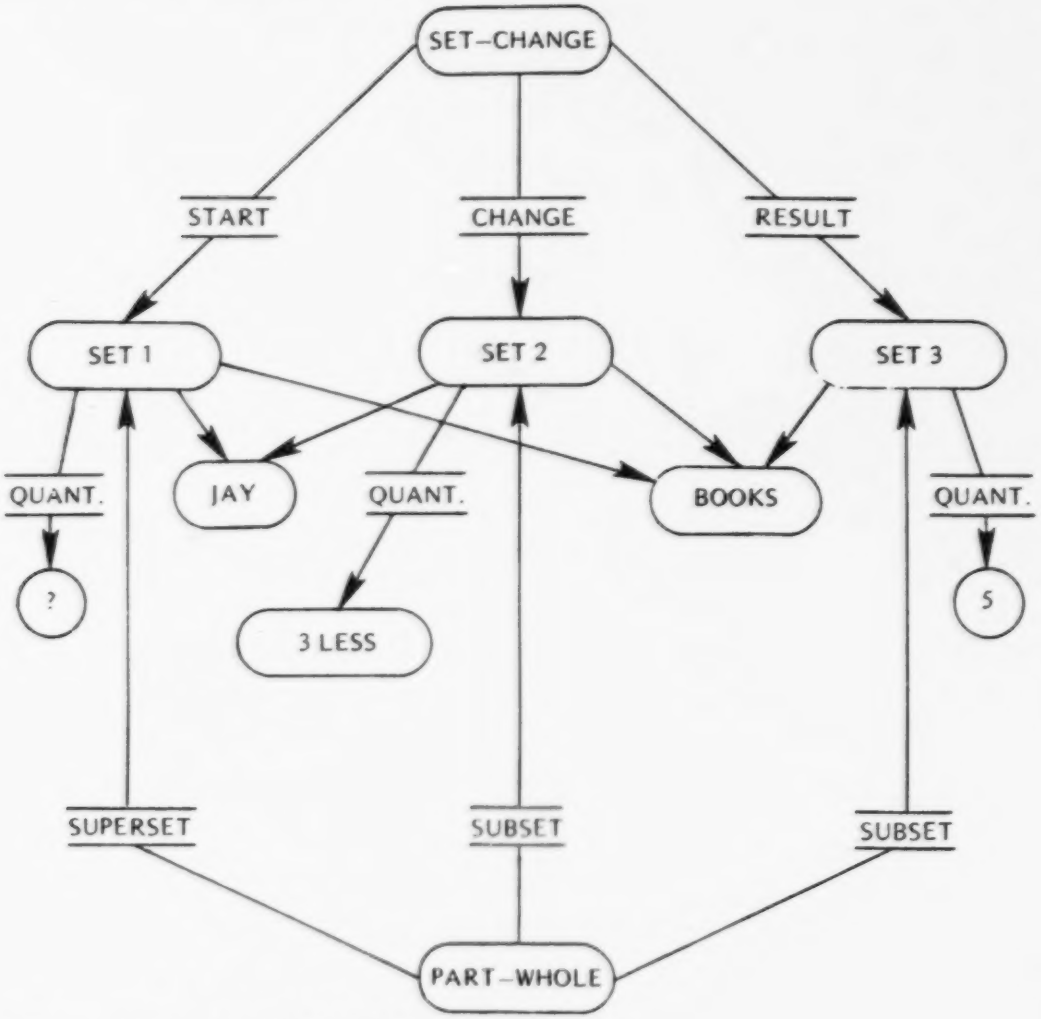

Figure 4: Schema-Based Representation of Problem (4)

In this case, the start set is the superset and the change and result are subsets. The final representation is shown in Figure 4. The model finds the answer by counting out objects to form the two subsets and then counts the total number of objects.

PROPOSITIONAL MODEL (work backward)

"Now he has five"	Count and place five blocks
"He lost three"	Count and place three more blocks (opposite of take-away)
"How many in the beginning"	Count all the blocks

Figure 5: Solution of the Propositional Model to Problem (4)

The solution to problem (4) by the propositional model uses knowledge about relations between operations. The model includes knowledge that taking things away and putting things somewhere are opposites, and that to find a state that was present before an action, you can perform the opposite of the action that was performed. With this relatively complex knowledge about actions, the model uses "now he has five of them" to count out five blocks, then uses "then he lost three of them" to count out three more blocks (the opposite of the action specified in the proposition), and finds the answer by counting all the blocks.

These two models provide an interesting example of the tradeoff between complexity of information and complexity of procedures. The schema-based model constructs representations with more complex information, and needs less complex procedures to solve the problems. The propositional model has less information in its representations, and therefore requires more complex knowledge of procedures to obtain solutions.

We now return to the concept of competence. The two process models of word problem solving illustrate the point made earlier that process models provide important information about performance but lack representations of the understanding of general principles. We believe that the important mathematical principles involved in word problems are the principles of set theory: principles of cardinality, subsets, complementary sets, unions, disjoint sets, and set differences. Performance of the models is consistent with the principles, but there is no explicit representation in the models of the principles or how the principles relate to the representations and procedures.

Cardinality (number implies a set)

Subset, complementary sets

Union, disjoints

Set difference

Figure 6: Principles in Word Problems

In our analyses, we postulate competence of two kinds--linguistic competence and operational competence. Linguistic competence is understanding of principles related to forming representations. Operational competence is understanding of principles related to the actions that are performed to solve the problem.

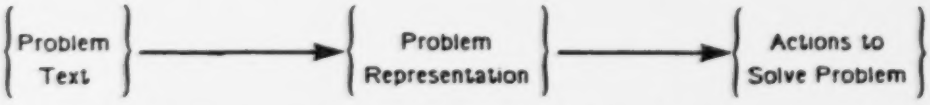

Figure 7: Components of Problem Solving Process

Figure 7 shows the main components of the problem-solving process. The three components here are the parts that are included in the process models, where problem texts are given as input, problem representations are constructed, and then problem-solving operators are used to find the solutions.

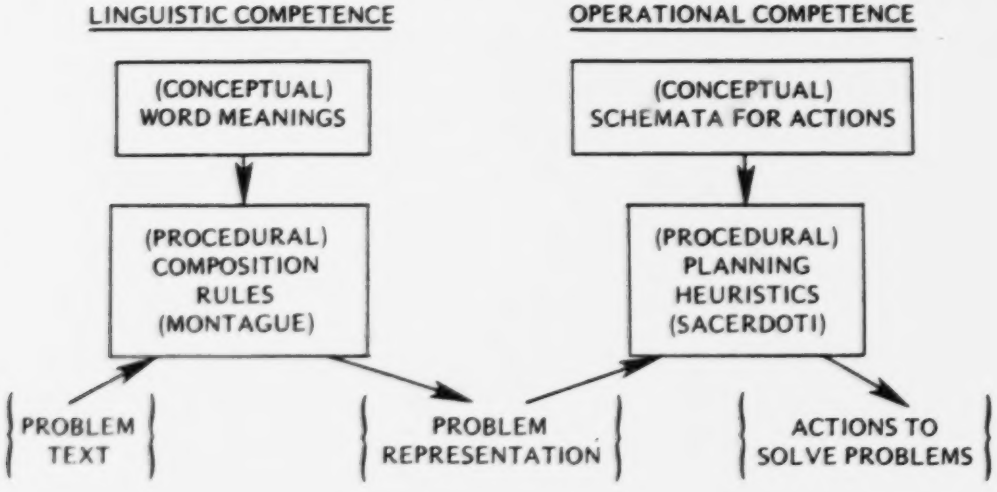

Figure 8: Components of Competence Analysis of Word Problem Solving

The additional components in Figure 8 correspond to the competence analysis.

LEARNING RESEARCH AND DEVELOPMENT CENTER

COMPETENCE FOR KNOWING AND UNDERSTANDING PROBLEMS

[illegible]17

JAMES G. GREENO and WALTER JOHNSON

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

- This document has been reproduced as received from the person or organization originating it.
- ☐ Minor changes have been made to improve reproduction quality.

- Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

As we said earlier, in the analysis of competence general principles are formulated in a way that allows their use in deriving properties of process models. The analysis requires at least two components: the principles and a system for the derivations. We refer to our formulation of the principles as conceptual competence and the derivational system as procedural competence.

In linguistic competence, the principles of set theory are included in knowledge of word meanings. We use the formal system of Montague grammar which provides a way to express word meanings and a set of rules for the composition of the meanings of words into meanings of phrases and sentences. These rules provide the system for deriving expressions based on the conceptual competence of word meanings.

In operational competence, principles that relate sets to procedures are represented as schemata for actions. These are general structures that incorporate relations between actions and conditions in ways that correspond to general principles of counting and relations between sets and numbers. Derivations of procedures are obtained by using planning heuristics, which satisfy goals that are taken from the representations and plan a set of actions that satisfy the goals (Sacerdoti, 1977).

LINGUISTIC COMPETENCE

Stronger form	Weaker form
Numerals imply existence of sets; e.g.	
"Jay has eight books" $\exists(X)[8(X) \wedge \forall x(x \in X \rightarrow \underline{book}(x)) \wedge \forall x(x \in X \rightarrow \underline{has}(Jay,x))]$	"Jay has eight books" has (Jay, 8 (books))
"of" means subset; e.g.:	
"He lost three of them" $[\mathbb{P}[\exists Y[3(Y) \wedge \underline{subset}(Y,X) \wedge \forall y(y \in Y = \underline{lose}(Jay,y))]]$	"He lost three of them" lost (Jay, 3 (of (8 (books))))
"More" means complement	
"Altogether" means union	
"More than," "less than" means difference	

Figure 9: Two Forms of Linguistic Competence

We have formulated two versions of competence, corresponding to the two kinds of process models that have been developed. In one version, knowledge of set-theoretic principles is included in the linguistic competence. A principle of cardinality is expressed in the meanings of numerals. The expression for the meaning of a numeral includes an existential quantifier. The thing that exists is a set, so that a term such as "eight" means that there is a set with cardinality 8. Principles of subset, complementary set, union, and so on, are also expressed in word meanings. For example, a subset relation is understood as the meaning of "of," so that "three of them" means that a set (with cardinality 3) is a subset of some previously mentioned set. A complement relation is understood as the meaning of "more" in a sentence such as "he found five more books." A union relation is understood as the meaning of "altogether," and a relation of set difference is understood as the meaning of "more than" and "less than." These assumptions of competence lead to representations like those of the schema-based process models that we described earlier.

In the other version of competence, meanings of numerals and other terms do not include set-theoretic properties and relations. For example, the meaning of a numeral is a kind of adjectival modifier that applies to the kind of thing that is mentioned, so for example, "eight books" is syntactically similar to "red books" (although of course their semantics are quite different). The representations that are derived are propositionally coherent but do not include the set-theoretic meanings of the terms.

OPERATIONAL COMPETENCE

Linguistics is Stronger:	Linguistics is Weaker:
Basic concepts of counting and number are different	Additional concepts reversibility, inclusion are needed for interpretation.

Figure 10: Two Forms of Operational Competence

We have also formulated two versions of operational competence, corresponding to the two versions of linguistic competence. Recall that operational competence represents principles as schemata for

actions, so that planning can be applied to obtain procedures. When linguistic competence is strong, operational competence can be quite simple. Only basic concepts of numbers and counting are needed, because the information about goals and sets of objects is already in the representations.

When linguistic competence is weak, a more complex version of operational competence is needed. The representations do not specifically refer to sets and their relations, so additional procedures must be produced to interpret the representations before counting can be performed. These additional procedures of interpretation require concepts such as the reversibility of actions, and inverse relations of inclusion and containment, familiar from the classic studies of Piaget.

The two analyses that we are developing could be considered as competing hypotheses about competence in the domain of quantitative language and problem solving. We do not view them that way. Instead, it seems reasonable to expect that the system with weaker linguistic competence represents understanding by young children, and as they develop and learn, they acquire components of the stronger linguistic competence. Indeed, there is evidence that supports this view. We have used the method introduced by Markman (1979) to investigate children's understanding of sets. Markman showed that when situations are described using class terms (e.g., "trees" or "soldiers"), children perform less well in quantitative tasks than when descriptions include collection terms (e.g., "forest" or "army"). We interpret this as indicating that collection terms invite representations that include references to sets, which assist the children in arriving at correct conclusions. We have studied performance with word problems, varying the wording so that in one version there are only class terms, but in another version collection terms are used as well. In our preliminary data, this variable has a facilitating effect on performance of 6-year-old children, but not for 7- or 8-year-old children. A reasonable conjecture is that understanding of the number-set relation is still problematic for the younger children, but not for the older ones, whose difficulties are caused by other factors of competence.

Discussion

The problem raised initially is the relation between cognitive processes and understanding of general principles. The question can be put: "What do process models tell us about what students (or other people) really understand?"

We have described the beginning of an attempt to answer that question. We have analyzed a task

for which process models have been developed, and we have found ways to relate those process models to formulations of general principles that are reflected in the processes.

We mentioned earlier that our analysis follows that of Chomsky in the domain of language syntax. Like Chomsky, we have characterized implicit understanding, which he called competence, in a way that allows the derivation of specific performance. In Chomsky's system, the derivations produce sentences of a language that are consistent with its grammar. In our system, one set of derivations produce representations with meanings that are consistent with set-theoretic principles; the other derivations produce procedures that are set-theoretically correct. We are hopeful that this kind of analysis will provide a valuable new dimension in the analysis of problem solving. The initial results seem promising.

References

Anderson, J. R. (1983). *The architecture of cognition*. Cambridge, MA: Harvard University Press.

Briars, D., & Larkin, J. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction, 1*, 57-84.

Brown, J. S., & Burton, R. R. (1978). Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science, 2*, 155-192.

Chomsky, N. (1965). *Aspects of a theory of syntax*. Cambridge, MA: MIT Press.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Gelman R. & Meck E. (1983). Preschooler's counting: Principles before skill or skill before principles? *Cognition, 13*, 343-359.

Greeno, J. G. (1978). A study of problem-solving. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 1). Hillsdale, NJ: Erlbaum.

Kintsch, W., & Greeno, J. G. (1984). Understanding and solving arithmetic word problems. *Psychological Review, 92*, 109-129.

Larkin, J. H., McDermott, J., Simon, D. P., & Simon, H. A. (1980). Models of competence in solving physics problems. *Cognitive Science, 4*, 317-344.

Markman, E. M. (1979). Classes and collections: Conceptual organizations and numerical abilities. *Cognitive Psychology, 11*, 395-411.

Resnick, L. (1983). A developmental theory of number understanding. In H. P. Ginsberg (Ed.), *The development of mathematical thinking*. New York: Academic Press.

Riley, M. S., Greeno, J. G., & Heller, J. I. (1983). Development of children's problem solving ability in arithmetic. In H. P. Ginsberg (Ed.), *The development of mathematical thinking*. New York: Academic Press.

Sacerdoti, E. D. (1977). *A structure for plans and behavior*. New York: North-Holland.